30/12/2023

Printing Pages :2

Paper Code: M.Sc.-Math-105 A (SVSU:2023-24/R)

Enrollment No.													

Program Name: M.Sc.-Math

I-Semester / I-Year Examination

Subject Name: Calculus of Variations

[Time : 03:00 Hours] **[Max. Marks : 70]**

Note: 1. Attempt all the questions as per given instructions.

1. **Multiple Choice Questions (MCQs)** **[01×15=15]**

I The shortest curve joining two fixed points is
(a) Any curve (b) Circle (c) Straight line (d) All

II The order of the differential equation xdy = ydx is
(a) 1 (b) 2 (c) 3 (d) 4

III In the functional, if $\delta^2 I > 0$ for every variation $\delta I\ (\neq 0)$, then the functional I(y) is
(a) maximize (b) minimize (c) Stable (d) None

IV The solution of the differential xdx = ydy is
(a) $x^2+y=0$ (b) $x^2+y^2=c$ (c) $x^2-y^2=c$ (d) None

V The degree of the differential equation xdy = ydx is
(a) 1 (b) 2 (c) 3 (d) 4

VI The solution of the differential xdx = -ydy is
(a) $x^2+y=0$ (b) $x^2+y^2=c$ (c) $x^2-y^2=c$ (d) None

VII Every continuous function is
(a) Differentiable (b) Not differentiable
(c) both (a) & (b) (d) has infinite limit

VIII The function f(x) = c is
(a) bounded (b) unbounded (c) may be both (d) None

IX The full form of COV is
(a) Complex of variation (b) Calculus of variation
(c) Co-homology of variation (d) None

X In the functional, if $\delta^2 I < 0$ for every variation $\delta I\ (\neq 0)$, then the functional I(y) is
(a) maximize (b) minimize (c) Stable (d) None

XI A necessary condition that the integral $\int_{x_1}^{x_2} F(x, y, y')\,dx$ will be stationary if it's
(a) last variation vanishes (b) Variation never be vanishes
(c) Second variation vanishes (d) First variation vanishes

XII If y(x) is the extremal of the functional $\int_{x_1}^{x_2} F(x, y, y')\,dx$ it satisfies

(a) $\frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y}\right) = 0$ (b) $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

(c) $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y}\right) = 0$ (d) $\frac{d}{dx}\left(\frac{\partial F}{\partial y}\right) - \frac{\partial F}{\partial y'} = 0$

XIII In an integral $\int_{x_1}^{x_2} F(x, y, y')dx$ if F is explicitly independent of x, then the Euler Lagrange equation is

(a) $F - y' = c$ (b) $F - y'\left(\frac{\partial F}{\partial y}\right) = c$ (c) $F - y'\left(\frac{\partial F}{\partial y'}\right) = c$ (d) $F = c$

XIV A necessary condition that the integral of a functional will be stationary if

(a) $\partial I = 0$ (b) $\partial I = c$ (c) $\partial I \neq 0$ (d) $\partial I \neq c$

XV The extremizing curve of the Brachistochrone problem is a

(a) Circle (b) Catenary (c) Cycloid (d) Straight line

2. Answer in long (any three) [10×3=30]

I Prove the Legendre condition for extremum.

II Show that the area of the surface of revolution of a curve $y = y(x)$ is $2\pi\int_{x_1}^{x_2} y\sqrt{(1+y'^2)}dx$. Hence show that for this to be minimum, the curve must be catenary.

III State and prove Euler's equation.

IV Show that the Jacobi condition for the central field of extremals for $I(y) = \int_0^{\pi/2}\left(xyy'+y^2-\frac{y'^2}{2}\right)dx$; $u(0) = 0$, where $u = \partial y$ is fulfilled. Also, show that for the extremals the functional is maximum.

V Prove that the extremal of the isoperimetric problem $I[y(x)] = \int_1^4 y'^2 dx$ with $y(1) = 3$, $y(4) = 24$ subject to the condition $\int_1^4 ydx = 36$ is a parabola.

3. Answer in short (any five) [5x5=25]

I Find the minimum distance between circle $x^2+y^2 = 1$ and straight line $x+y = 4$.

II Solve the boundary value problem $y'' - y+x = 0\ (0 \le x \le 1)$, $y(0) = y(1) = 0$ by Rayleigh-Ritz method.

III Find the plane curve of fixed perimeter and maximum area.

IV Find the shortest distance between parabola $y^2 = 4x$ and $(x-9)^2 + y^2 = 4$.

V Using variational method, show that the shortest curve joining two fixed points is a straight line.

VI Prove the Lagrange's equation.

VII Find the extremals of the functional $\int_{x_1}^{x_2}(1+x^2y')y'dx$.

VIII Find the extremals of the isoperimetric problem $\int_{x_0}^{x_1} y'^2 dx$ given that $\int_{x_0}^{x_1} ydx = c$, a constant.

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Paper Code: M.Sc.-Math-102 A (SVSU:2023-24/R)

Enrollment No.													

M.Sc. MATHEMATICS

Semester I /Year I Examination

Subject Code - M.Sc. MATH 102

Real Analysis

[Time : 3:00 Hours] **[Max. Marks : 70]**

Note: 1. Attempt all the questions as per given instructions.

I Objectives Question: Answer All Question **[15×1=15]**

1. The integral $\int_0^\infty \frac{dx}{x^n}$ where a> 0 *is convergent when*
   (a) n= 1 (b) n< 1 (c) $n \leq 1$ (d) n> 1
2. Every continuous function defined on (a, b) is
   (a) Not Riemann integrable (b) Riemann integrable
   (c) proper integrable (d) improper integrable
3. Supremum is
   (a) greatest lower bound (b) least upper bound
   (c) both a and b (d) none of these
4. The integral $\int_3^\infty \frac{dx}{(x-2)^2}$ is
   (a) Divergent (b) Convergent (c) Both a and b (d) none of these
5. The series $\sum \frac{1}{n^p}$ is convergent if
   (a) P< 1 (b) P=1 (c) P> 1 (d) none of these
6. The series $\sum \frac{1}{n^{2/3}}$ is
   (a) Divergent (b) Convergent (c) oscillatory (d) none of these
7. The series $\sum_1^\infty \cos\frac{1}{n}$
   (a) Divergent (b) Convergent (c) oscillatory (d) none of these
8. The series $\sum_1^\infty \sin\frac{1}{n}$
   (a) Divergent (b) Convergent (c) oscillatory (d) none of these
9. The integral $\int_0^\infty \frac{\cos x}{1+x^2}dx$
   (a) Divergent (b) Convergent (c) oscillatory (d) none of these
10. The integral $\int_0^1 \frac{dx}{(x)^{\frac{1}{2}}}$
    (a) Divergent (b) Convergent (c) oscillatory (d) none of these
11. The auxiliary series, $\sum \frac{1}{n^p}$ is divergent if:
    (a)p = 1 (b) p≤ 1 (c) p≥ 1 (d) p> 1 .
12. If $\sum u_n$ be a series of positive terms such that, $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = l$. If $l=$ 1, then series is:
    (a) convergent (b) divergent (c)oscillatory (d) test fail.

P.T.O.

13 If $\sum u_n$ be a series of positive terms such that, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$. If $l < 1$, then series is:

(a) convergent (b) divergent
(c)oscillatory (d) may either converge or diverge.

14 If $\sum u_n$ be a series of positive terms such that, $\lim_{n\to\infty} \left\{n\left(\frac{u_{n+1}}{u_n} - 1\right)\right\} = l$. If $l > 1$, then series is:

(a) convergent (b) divergent
(c)oscillatory (d) may either converge or diverge

15 The series, $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots \dots \dots + \frac{1}{n(n+1)} + \dots \dots \dots$ is called:

(a) finite series (b) Alternating series
(c) telescoping series (d) negative terms series .

II. Long Question: Answer any Three **[10×3=30]**

1. if $u = ax^2 + by^2 + cz^2$ where $x^2 + y^2 + z^2 = 1$ and $lx+my+nz=0$ prove that stationary values of u satisfy the equations $\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0$
2. Find the expansion for cos x cos y in powers of x, y up to fourth order term
3. if $f(x,y) = \begin{pmatrix} \frac{xy}{\sqrt{x^2+y^2}} & when\ (x,y) \neq (0,0) \\ 0 & when\ (x,y) = (0,0) \end{pmatrix}$
   Show that f is continuous at (0,0)
4. State and prove Abel's test for Uniformly convergent
5. State and prove Necessary and sufficient condition for RS - Integrability

III. Short Question: Answer any Five **[5×5=25]**

1. Define Riemann Stieltjes integral
2. State and prove Darboux Theorem
3. Define Uniform convergence of sequences
4. Show that the sequences $< f_n >$ where $f_n(x) = nx(1-x)^n$ does not converge uniformly on [a,b]
5. Define power series with example
6. Find the radius of convergence of power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$
7. Show that $\lim_{(x,y)\to(0,0)} \frac{2x^3-y^3}{x^2+y^2} = 0$
8. Find the maximum and minimum values of the function $x^3y^2(1-x-y)$
9. Write the Cauchy's General Principle of uniform Convergence
10. State and prove Fundamental theorem of integral Calculus.

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Printing Pages : 3

Paper Code :MSc-Math-104 C (SVSU:2022-23/R)

Enrollment No.													

Program Name : M.Sc.-MATHEMATICS

I- Semester / I -Year Examination

Subject Name: Mathematical Methods -Code: MSc-MATH-104

[Time : 03:00 Hours] **[Max. Marks : 70]**

Note : Attempt all the questions as per given instructions.

1. Multiple Choice Questions (MCQs) **[01×15=15]**

I The Volterra Linear Integral Equation contains

(a) all limits are constant, (b) all limits are variable,

(c) one of the limits of integration is a variable (d) all.

II The integral equation α (x) u(x) = F(x) + $\mu \int_a^b K(x,\xi)$ u(t)dt is a

(a)Volterra integral equation, (b) Fredholm integral equation,

(c) both, (d) none

III When $\alpha = 0$, then the integral equation F(x) = $\mu \int_a^b K(x,\xi)$u(t)dt is a

(a) First kind Volterra integral equation,

(b) First kind Fredholm integral equation,

(c) Second kind Volterra integral equation

(d) Second kind Fredholm integral equation,

IV When $\alpha = 1$,then the integral equation : u(x) = F(x) + $\mu \int_a^b K(x,\xi)$ u(t)dt is a

(a)Second kind Volterra integral equation,

(b) First kind Fredholm integral equation,

(c) First kind Volterra Integral equation,

(d) Second kind Fredholm integral equation.

V When $\alpha = 1$ and F(x) = 0 , then the integral equation :

u(x) = $\mu \int_a^b K(x,\xi)$ u(t)dt is a

(a) Second kind Volterra integral equation,

(b) First kind Fredholm integral equation,

(c) First kind Volterra Integral equation,

(d) Second kind Fredholm integral equation.

VI The root – mean square value of a function y = f(x) over an internal from x = a to x = b is defined as

(a) $\bar{y} = 1/(b-a)^{1/2} \int_a^b y^2\,dx$ (b) $\bar{y} = 1/(b-a)^{1/2} \int_a^b y\,dx$

(c) $\bar{y} = 1/(b-a) \int_a^b y^2 dx$ (d) $\bar{y} = [\,1/(b-a) \int_a^b y^2\,dx]^{1/2}$

VII The Parseval's Theorem for Fourier Series is

(a) $\bar{y}^2 = a_0^2/2 + 1/2\sum(a_n^2 + b_n^2)$ (b) $\bar{y}^2 = a_0^2/4 + 1/2\sum(a_n^2 + b_n^2)$

(c) $\bar{y}^2 = a_0^2 + 1/2\sum(a_n^2 + b_n^2)$ (d) $\bar{y}^2 = a_0^2/2 + \sum(a_n^2 + b_n^2)$.

VIII Which is not a possible solution one dimensional heat equation:
(a) $u(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})Ce^{\alpha\lambda t}$ (b) $u(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})Ce^{-\alpha\lambda t}$
(c) $(u(x,t) = (Ae^{\lambda x} - Be^{-\lambda x})Ce^{-\alpha\lambda t}$ (d) $u(x,t) = (Ax + B)C$

IX Find the constant term of the Fourier Series of $f(x)= x^2$ in $(-\pi,\pi)$ is
(a) π (b) π^2 (c) $2\pi^2/3$ (d) $\pi^2/3$.

X The value of the a_0 of the Fourier Series of $f(x)= x + x^2$ in $(-\pi,\pi)$ is
(a) π (b) π^2 (c) $\pi^2 // 2$ (d) $2\pi^2/3$.

XI The integral equation $u(x) = \mu \int_0^x K(x,\xi)\, u(t)dt$ is a
(a) Second kind Volterra integral equation (b) Fredholm integral equation (c) Volterra Integral equation (d) none

XII The integral equation $\emptyset(x)u(x) = F(x) + \mu \int_0^x K(x,\xi)\, u(t)dt$ is a
(a) Third kind Volterra integral equation (b) Fredholm integral equation (c) Volterra Integral equation (d) non

XIII The integral equation $u(x) = F(x) + \mu \int_a^x K(x,\xi)\, u^n(t)dt$, $n>1$, is a
(a) Second kind Volterra integral equation (b) Fredholm integral equation (c) Volterra non-Integral equation (d) none

XIV The integral equation $u(x) = F(x) + \mu \int_a^b K(x,\xi)\, u^n(t)dt$, $n>1$, is a
(a) Second kind Volterra integral equation (b) Fredholm integral equation (c) Frsdholm non-Integral equation (d) none.

XV The integral equation $u(x) = F(x) + \mu \int_a^x \emptyset[x,\xi,u(\xi)]\, u^n(t)dt$, $n>1$, is a
(a) Volterra integral equation (b) Fredholm integral equation
(c) Volterra non-Integral equation (d) none.

2. Answer in long (any three) **[10×3=30]**

I Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} - \sin x\frac{dy}{dx} + e^x y = x$ with the initial conditions $y(0)=1, y'(0)=-1$.

II Express $f(x)= (\pi-x)^2$ as a Fourier Series of periodicity 2π in $0<x<2\pi$ & hence deduce $\sum 1/n^2=\pi^2/6$.

III Solve the following integral equation:

$$\phi(x) = \left(\sin x - \frac{x}{4}\right) + \frac{1}{4}\int_0^{\pi/2} \xi x \phi(\xi)d\xi.$$

IV Obtain Fredholm integral equation of second kind corresponding to the boundary value problems $\frac{d^2\phi}{dx^2} + \lambda\phi = x; \phi(0) = 0, \phi'(1) = 0$. Also, recover the boundary value problem from the integral equation you obtain.

V Find the solution of $\left(\frac{\delta^2 u}{\delta x^2}\right) = h^2\left(\frac{\delta u}{\delta t}\right)$ for which

$U(0,t) = u(l,t) = 0, u(x,0) = \sin\frac{\pi x}{l}$ by the methods of variable separable.

3. Answer in short (any five) **[5x5=25]**

I Expand f(x) by Fourier series where f(x) is defined by $f(x) = e^{-x}$ in $[-\pi, \pi]$.

II Find the resolvent kernel for the following kernel: $k(x,\xi) = (1+x)(1-\xi); a = -1, b = 0.$

III Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0.$

IV Obtain the Fourier series of periodicity 2π for $f(x) = x$ in $-\pi < x < \pi$.

V Show that the function $y(x) = (1+x^2)^{-3/2}$ is a solution of the Volterra integral equation $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)dt.$

VI Obtain the solution of the wave equation $\left(\frac{\partial^2 u}{\partial t^2}\right) = c^2\left(\frac{\partial^2 u}{\partial x^2}\right)$ using the method of separation of variables.

VII Find is the value of $D(x, \xi, \mu)$ whose Kernels is $K(x, \xi) = x\xi$ for $a = 0$ and $b = 10$.

15/12/2023

Printing Pages :2

Paper Code: M.Sc.-Math-101 A (SVSU:2023-24/R)

Enrollment No.													

Program Name M.Sc. (Mathematics)
I Semester /I Year Examination
Subject Code MSc Math 101
Subject Name:- Algebra

[Time : 3:00 Hours] **[Max. Marks : 70]**

Note: 1. Attempt all the questions as per given instructions.

I Objectives Question: Answer All Question [15×1=15]

1. Select Ring from the following
   A. (R, +,.) B. (Z, -,.) C. (R*, +,.) D. (Q% +,.)
2. Characteristic of Ring
   A. 3 B. 2 C. 1 D. 0
3. Which of the following is an example of an Ideal of Ring *R*
   A. Z B.{0} C. *Q* D. 2Z
4. Which of the following is not a Prime ideal of *Z*
   A. 5Z B. Z2 C. 3Z D. 2Z
5. Which of the following is a prime field
   A. *Z* B. *R* C. Q D. *C*

6 Number of elements in the Galois group of p^{th} cyclotomic polynomial over Q is
   A. 2 B. *p* C. $p - 1$ D. $p + 1$

7 Number of ideals of a Field is
   A. 0 B. 2 C. 1 D. 3

8 Example of a principal ideal of *Z* is
   A. Z_3 B. Z_4 C. *Q* D. 2*Z*

9 Find Galois group of the polynomial $x^2 - 3$ over Q
   A. Z2 B. Z3 C. *R*. D. *Z5*

10 -Which of the following is not a Fermat prime
   A. 3 B. 5 C. 17 D. 8

11 Which of the following is a cyclic group
   A. *Z3* x *Z3* B.*Z3* x *Z9* C. *Z2* x *Z4* D. *Z3* x *Z5*

12 Number of identity elements in a Ring is
   A. 2 B. 1 C. 3 D. 4

13 Number of units in the Integral Domain *Z*
   A. 1 B. 3 C. 2 D. 0

14 The number of normal subgroups in a nontrivial simple group
   A. 2 B. 1 C. 0 D. 4

15 Number of elements of order 5 in S4 is
   A. 1 B. 0 C. 5 D. 4

P.T.O.

II. Long Question: Answer any Three [3×10=30]

1. Show that ring of Gaussian integers is a Euclidean ring
2. State and prove Sylow's first theorem
3. If S be an ideal of a ring Z of all integers .Then S is maximal if and only if it is generated by Some prime integer
4. State and prove Unique factorization theorem

III. Short Question: Answer any Five [5×5=25]

1. Define Prime And Maximal ideals with example.
2. Define embedding of Ring.
3. Show that Every ring can be embedded in a ring with identity.
4. Show that any product of solvable group is solvable.
5. Show that the ring I of all integers is a Euclidean ring.
6. Define Galois Group with example.
7. Show that a field has no proper ideals.

Printing Pages :3

Paper Code :- MSc-Math-103 A (SVSU:2022-23/R)

Enrollment No.													

Program Name: M.Sc. (Maths)
Semester / Year: I/I Examination
Subject Code: MSc-Maths-103
Subject Name : Differential Equation

[Time : 03:00 Hours] [Max. Marks : 70]

Note : 1. Attempt all the questions as per given instructions.

1.	Multiple Choice Questions (MCQs)	[01×15=15]
I	The roots or the differential equation d2y/dx2+4y=0 is a) real and equal b) real and distinct c) imaginary d) none of these	
II	In Lagrange's equation P, Q and R are the functions of a) x and y b) x, y and z c) p, q and r d) x and z	
III	Equation p tan y + q tan x =sec^2 z is of order a) 1 b) 0 c) 2 c) none of these	
IV	The partial differential equation y(r) + x(t) =0 is hyperbolic in a) The second and fourth quadrants b) The first and second quadrant c) The second and third quadrants d) The first and third quadrant	
V	In initial value problem, the initial conditions are known as a) Cauchy conditions b) Dirichlet conditions c) Neumann conditions d) Robin conditions	
VI	a system of geometrical surfaces described by the equation a) f (x,y,z,p,q)=0 b) f (x,y,z,a,b)=0 c) f (x,y,a,b)=0 d) f (x,y,z,p,q,r,s,t)=0	
VII	A solution of differential equation of three order has a) 2 constants b) 3 constants c) 4 constants d) 1 constants	
VIII	A particular solution of differential equation involve only a) arbitrary constant b) not contain particular value of a arbitrary constant c) particular value of a arbitrary constant d) none of these	
IX	A function is called .... if all of its derivatives exist and are continuous. a) smooth b) weak c) strong d) none of these	
X	the envelope is called the singular solution of equation F(x, y, z, p, q) = 0 a) general solution b) singular solution c) particular integral d) complete integral	

XI The equation F(x, y, z, p, q) = 0 is said to be .... if F is linear in each of the variables z, p and q, and the coefficients of these variables are functions only of the independent variables x and y

a) linear partial differential equation
b) quasi-linear partial differential equation
c) semi-linear partial differential equation
d) non-linear partial differential equation

XII What is the solution of the following equation p+q=1

a) f(x-z2,y-z2)=0 b) f(x3-z,y3-z)=0
c) f(x-z,y-z)=0 d) none of these

XIII Which of the following equation is Clairaut's equation

a) x=py+f(p) b) y=px+fc c) y=px+f(p) d) y=x2+c

XIV The Particular integral of the differential equation(D2+1)y=0 is

a) y=c cos ax + d sin ax b) 1/2.sinx
c) zero d) none of these

XV In equation of the form dy/dx+Py=Q is called

a) variable separable b) exact
c) linear differential equation d) all of the above

2. Answer in long (any three) [10×3=30]

I Find the adjoint of the equation $x^2\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}+3y=0$

II Find the solution of two-dimensional Laplace's equation by using the method of separation of variables.

III Reduce the following equation to canonical form

$$\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$$

IV Show that the Wronskian of the function $x^a, x^b, \& x^c$ is equal to $(a-b)(b-c)(c-a)x^{a+b+c-3}$

3. Answer in short (any five) [5x5=25]

I State and prove the integral formula for confluent hyper geometric function ${}_1F_1(\alpha;y,x)$

II Evaluate $\int_0^\infty x^2 e^{-x^2}dx$

III Solve $x^2 p + y^2 q = z^2$

IV Form a partial differential equation by eliminating the parameter a and b from $z = axe^y + \frac{1}{2}a^2 e^{2y} + b$

V Find the complete integral of $z = px + qy + p^2 + q^2$

VI Solve $\left(D^3 - 4D^2 D' + 4DD'^2\right)z = 4\sin(2x + y)$

VII Classify the operator $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2}$