

Printing Pages :2

Paper Code: M.Sc.-Math-302 B (SVSU:2023-24/R)

Enrollment No.

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Program Name M.Sc. (Mathematics)

III Semester / II Year Examination

Subject Code MSc Math 302

Subject Name Measure & Integration

[Time : 3:00 Hours]

[Max. Marks : 70]

Note: 1. Attempt all the questions as per given instructions.

I Objectives Question: Answer All Question [15×1=15]

1. m^* is
(a) additive, (b) multi adaptive (c) sub additive (d) linear.
2. Cantor's ternary set is
(a) Countable (b) finite (c) uncountable (d) none
3. Sum of two simple function is a
(a) Step function (b) simple (c) constant (d) both (b) & (c)
4. If f is measurable on a measurable set E , and if $|f|$ is Lebesgue integrable then
(a) f is Riemann integrable, (b) f is Lebesgue integrable,
(c) both(a) &(b), (d) none
5. Zero set is
(a)Countable, (b) uncountable, (c) both (a)&(b) (d) none.
6. Every measurable function is
(a)Nearly a finite union of intervals, (b) continuous,
(c) discontinuous, (d) differentiable
7. If A is measurable then Characteristic function φ_A of a set A is
(a)countable, (b) measurable, (c) non-measurable, (d) all is correct
8. Constant function is
(a)not continuous, (b) measurable, (c) non-measurable, (d) all is correct
9. The set real numbers is
(a)Countable, (b) finite, (c) uncountable, (d) none.
10. Finite set always
(a)Countable, (b) equivalent to N , (c) uncountable, (d) none
11. If B is a countable subset of an uncountable set A , then $A-B$ is
(a) Countable, (b) infinite, (c) uncountable, (d) finite.
12. The measure of empty set is
(a)1, (b) -1, (c) 0, (d) 2.
13. The measure of the interval $[a,b]$ on real line is
(a) $a-b$, (b) $b-a$, (c) both, (d) none.
14. For every set A
(a) $m^*(A) \leq 0$, (b) $m^*(A) = 1$, (c) $m^*(A)$ is unique, (d) all is correct

- 15 The set numbers is
(a) Countable (b) finite (c) uncountable (d) none.

II. Long Question: Answer any Three [10x3=30]

1. State and prove F. Riesz theorem for Lebesgue measurable.
2. Prove that every bounded measurable function f defined on $[a,b]$ is Lebesgue integrable over $[a,b]$.
3. Show that cantor is uncountable
4. If $\{A_1, A_2, \dots\}$ is a countable family of subsets of R , then $m^*(\bigcup_{n=1}^{\infty} A_n) \leq \sum_{n=1}^{\infty} m^*(A_n)$.
5. State and prove Lebesgue Monotone convergence Theorem

III. Short Question: Answer any Five [5x5=25]

1. Show that every countable set is L - measurable and its measure is zero
2. If f is measurable function on a measurable set E , then prove that f is Lebesgue integrable if $|f|$ is Lebesgue integrable.
3. A sequence $\langle f_n \rangle$ which converges to f in measure is a Cauchy sequence in measure
4. If A and B are any two disjoint subsets of R , then $m^*(A \cup B) = m^*(A) + m^*(B)$.
5. Show that m^* is countably sub additive.
6. Prove that characteristic function of a set A is measurable iff A is measurable
7. Show that every bounded open set and every bounded closed set is measurable
8. State and prove Schorder-Bernstein theorem for cardinality of sets.
9. if $E_1, E_2, \dots$ are measurable sets then the set $\bigcap_{k=1}^{\infty} E_k$ is measurable
10. Define cantor ternary set

Printing Pages : 2

Paper Code : MSc-Math-304 C (SVSU:2022-23/R)

Enrollment No.

M.Sc. Mathematics

IInd YEAR EXAMINATION

Optimization Techniques(M.Sc. Math 304)

[Time : 3:00 Hours]

[Max. Marks : 70]

Attempt All Questions.

I. Objectives: Answer All Questions

[15×1=15]

1. Minimum principle is based on :
 - a) Concepts of calculus of variations
 - b) Principle of calculus
 - c) Principle of invariant imbedding
 - d) Principle of optimality
2. Which of the following problems is NOT solved using dynamic programming?
 - a) 0/1 knapsack problem
 - b) Matrix chain multiplication problem
 - c) Edit distance problem
 - d) Fractional knapsack problem
3. If a complex number z , with integral real and imaginary parts, satisfies $z^2 + \bar{z}^2 = 16$, then find the value of $|z|$.
 - a) $2\frac{1}{2}$
 - b) 4
 - c) $8\frac{1}{2}$
 - d) $10\frac{1}{2}$
4. Bellman's principal is used for?
 - (a) probabilistic programming
 - (b) stochastic programming
 - (c) dynamic programming
 - (d) None
5. For a complex number z , real numbers may have
 - a) 1
 - b) 2
 - c) 3
 - d) ≥ 2
6. Which of the following problems should be solved using dynamic programming?
 - a) Mergesort
 - b) Binary search
 - c) Longest common subsequence
 - d) Quicksort
7. If a problem can be solved by combining optimal solutions to non-overlapping problems, the strategy is called _____.
 - a) Dynamic programming
 - b) Greedy
 - c) Divide and conquer
 - d) Recursion
8. If the complex numbers $(x^2 - 3x + 2) + i(y^2 - 11y + 40)$ and $(x^2 - 6x + 8) + i(y^2 - 9x + 10)$ are conjugates of each other, Then what is the value of $|x + iy|$?
 - a) $13\frac{1}{2}$
 - b) $19\frac{1}{2}$
 - c) $23\frac{1}{2}$
 - d) $29\frac{1}{2}$
9. In dynamic programming, the technique of storing the previously calculated values is called _____.
 - a) Saving value property
 - b) Storing value property
 - c) Memoization
 - d) Mapping
10. Which of the following is/are property/properties of a dynamic programming problem?
 - a) Optimal substructure
 - b) Overlapping subproblems
 - c) Greedy approach
 - d) Both (a) and (b)

11. Let $z = \sin x + i \cos 2x$ and $\omega = \cos x - i \sin 2x$. Then for what values of x are z and ω conjugate of each other?
 a) $x = n\pi$ b) $x = 0$ c) $x = (n+1/2)\pi$ d) no value of x
12. Which one of the following is false?
 (a) A real-valued function that is continuous on its measurable domain is measurable. (b) A monotone function that is defined on an interval is measurable. (c) A monotone function that is defined on an interval need not be measurable. (d) Let f be an extended measurable real-valued function on E and $f = g$ a.e. on E , then g is measurable on E .
13. Let z and ω be complex numbers satisfying $z\bar{z} + \omega\bar{\omega} = 100$. If $|z|, |\omega| \in I$, then find the minimum possible value of the expression $|z + \omega|$.
 a) 1 b) 2 c) 3 d) 4
14. For two complex numbers p and q , if $\text{Arg}(p) - \text{Arg}(q) = \pi/2$ as well as $|pq| = 1$, what is the value of $\bar{p}q$?
 a) -1 b) $-i$ c) i d) 1
15. When dynamic programming is applied to a problem, it takes far less time as compared to other methods that don't take advantage of overlapping sub problems.
 (a) True (b) False (c) both (d) None

II Long Question : Answer Any Three

[10×3=30]

- Describe Dynamic programming for the performance index and their applications.
- What do you understand by Bellman's principle of optimality? Explain its uses.
- Consider the extended real valued function $y = f(x) = 1/x^2$ where $x \in [-1, 1]$. Show that for any $0 < \epsilon < 1$
- Describe optimal substructure and overlapping subproblems.
- Discuss directional derivative and its uses.

III. Short Question: Answer any Five

[5×5=25]

- What are real functions and real-valued functions?
- What do you understand by Measurable functions?
- Discuss Duality and optimality for standard convex programs.
- Write the Hamilton-Jacobi-Bellman equation.
- Discuss an Analytic function?
- Write a short note on 'Stage coach problem'.
- Define Gradient descent method.
- Write a short note on performance index.

MSC-MATHEMATICS
III - YEAR EXAMINATION
Discrete Mathematics
MSC--MATH-308

[Time : 3:00 Hours]

[Max. Marks : 70]

I. Objectives Question: Answer All Question

[01×15 = 15]

1. There is no horizontal line in a Hasse diagram of a poset
 (a) False (b) true (c) both (i) and (ii) (d) none
2. The list of all of the maxterms of the two variables x and y are
 (a) $x + y, x' + y, x + y', x' + y'$ (b) $x + y, x', x + y', x + y'$
 (c) $xy, x' + y, y', x + y'$ (d) $x + y, x + y', x + y', x'$
3. In Boolean Algebra, $a \cdot (a + b)$ is equal to:
 (a) b (b) a (c) 1 (d) 0
4. In Boolean Algebra, aa' is equal to
 (a) b (b) a (c) 1 (d) 0
5. In Boolean Algebra, $a + a'$ is equal to :
 (a) 2a (b) a (c) 1 (d) 0
6. In Boolean Algebra, $a + (a \cdot b)$ is equal to:
 (a) b (b) a (c) 1 (d) 0
7. The output of a 2 input OR gate is 0 only when its
 (a) both input are 0 (b) either input is 0
 (c) both input are 1 (d) either input is 1
8. In Boolean Algebra, $a + a$ is equal to :
 (a) 2a (b) a (c) 1 (d) 0
9. In Boolean Algebra, $a \cdot (a + b \cdot (a + b))$ is equal to :
 (a) a (b) c (c) $a + b$ (d) $c \cdot b$
10. The sum of products form of $ab + bc'$ is:
 (a) $abc + abc' + a'bc'$ (b) $abc + abc' + a'b'c'$
 (c) $abc' + a'bc' + a'b'c'$ (d) $abc' + a'bc' + a'b'c'$
11. The maximum number of edges in a simple graph with n vertices is :
 (a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$
 (c) $\frac{(n-1)}{2}$ (d) $\frac{n(n-1)}{3}$
12. The minimum number of edges in a connected graph with n vertices is:
 (a) $n-1$ (b) $n+1$ (c) n (d) none of these
13. Any connected graph with n vertices and $(n-1)$ edges is a :
 (a) rooted tree (b) tree
 (c) binary tree (d) none of these

14. In a bounded distributive lattice if a complement exists it is
(a) unique (b) different (c) both (i) and (ii) (d) none

15. Every distributive lattice is
(a) Modular (b) not modular (c) both (i) and (ii) (d) none

II Long Question : Answer Any Three [10×3=30]

1. Draw the Hasse diagram from the divisibility relation on $\{2,4,5,10,12,20,25\}$. Starting from the digraph.

2. If $(L, \wedge, \vee)$ be a complemented and distributive lattice, then for any $a, b \in L$, prove that:
(i) $\overline{a \vee b} = \overline{a} \wedge \overline{b}$ (ii) $\overline{a \wedge b} = \overline{a} \vee \overline{b}$.

3. (i) In Boolean Algebra, show that:
 $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(a + b')(c' + a)$
(ii) Use Karnaugh map to simplify the function:
 $X = A'B'C' + A'B'C + A'BC + A'BC' + AB'C + ABC$

4. The State table of a finite state Machine M is given in:

f, g	a	b
S_0	S_0, b	S_4, b
S_1	S_0, a	S_3, b
S_2	S_0, a	S_2, a
S_3	S_1, b	S_1, b
S_4	S_1, b	S_0, a

(i) Find the input set I, the State set S, the output O and initial state of M

(ii) Draw the state diagram of M.

(iii) Find the Output of the word $w = a^2bab^2a$.

5. Explain Kruskal's algorithm on tree with an example.

III. Short Question: Answer any Five [5×5=25]

1. Define minset give an example.

2. Define Hasse diagram with an example.

3. Define Lattices with example.

4. For every element a in a Boolean Algebra B.
then prove that: $a + a = a$ and $a.a = a$.

5. Show that compliment of every element of $a \in B$ is unique.

6. Use Karnaugh map to simplify the: $X = ABC' + ABC$.

7. Prove that a graph G with n vertices is a tree if and only if it has n-1 edges and no cycles.

8. Draw the multi-graph G corresponding to each of the following adjacency matrices:

$$(a) A = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad (b) A = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

9. Explain tautology and contradiction with examples.

10. Define derived connectives NAND, NOR and XOR gates with the help of truth table.

Printing Pages :3

Paper Code :- MSc-Math-303 B (SVSU:2022-23/R)

Enrollment No.

Program Name: M.Sc.-Math

III-Semester / II-Year Examination

Subject Name: Numerical Analysis- Code: MSc-Math-303

[Time : 03:00 Hours]

[Max. Marks : 70]

Note: Attempt all the questions as per given instructions.

1. Multiple Choice Questions (MCQs)

[01×15=15]

I In the general Newton-Raphson formula is

$$(a) x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)} \quad (b) x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$$

$$(c) x_{n+1} = x_n - \frac{f(x_n)}{f''(x_n)} \quad (d) x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

II Newton-Gregory Forward interpolation formula can be used

- (a) only for equally intervals (b) only for unequally intervals
(c) for both equally and unequally intervals
(d) for unequally intervals

III What is the percentage decrease in an interval containing root after iteration is applied by Bisection Method?

- (a) 20% (b) 30% (c) 40% (d) 50%

IV The value of $\Delta f(x)$, where Δ is forward difference operator and h is interval

- (a) $f(x+h) - f(x)$ (b) $f(x-h) + f(x)$ (c) $f(x-h) - f(x)$ (d) $f(x) - f(x-h)$

V The Lagrange's interpolation formula is used for.....spaced value of x .

- (a) equal (b) unequal (c) neither (a) and nor (b) (d) both (a) and (b)

VI In Simpson's one third (1/3) rule, formula can be used if

- (a) n is even (b) n is integer
(c) n is a multiple of 3 (d) n is a multiple 6

VII In Simpson's third eight (3/8) rule, formula can be used if

- (a) n is even (b) n is integer
(c) n is a multiple of 3 (d) n is a multiple 6

VIII In Simpson's one third (1/3) rule, $f(x)$ is a polynomial of

- (a) one degree (b) second degree (c) third degree (d) fourth degree

IX In Simpson's third eight (3/8) rule, $f(x)$ is a polynomial of

- (a) one degree (b) second degree (c) third degree (d) fourth degree

- X In Trapezoidal rule, $f(x)$ is a
 (a) linear function (b) non-linear function
 (c) polynomial (d) none of these
- XI In trapezoidal rule, the value of $\int_a^b y dx = \dots$
 (a) $\frac{1}{2}(a+b)$ (b) $\frac{h}{2}(a+b)$ (c) $\frac{h}{4}(a+b)$ (d) $\frac{h}{3}(a+b)$
- XII The Runge-Kutta method of third order is equal to
 (a) $y = y_0 + \frac{1}{2}(k_1 + k_2)$ (b) $y = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
 (c) $y = y_0 + \frac{1}{3}(k_1 + 4k_4 + k_3)$ (d) $y = y_0 + \frac{1}{6}(k_1 + 4k_4 + k_3)$
- XIII Which method is faster than Gauss-Jacobi
 (a) Gauss-Seidel (b) Crout's method
 (c) Picard's method (d) Euler's method
- XIV Newton-Raphson method fails if
 (a) $f'(x) = 0$ (b) $f''(x) = 0$ (c) $f(x) = 0$ (d) $f'''(x) = 0$
- XV Which method is called Bolzano method?
 (a) False position method (b) Newton-Raphson method
 (c) Secant method (d) Bisection method
2. Answer in long (any three) [10×3=30]
- I Find the solution of the system of equations by Gauss-Seidel method
 $28x + 4y - z = 32$, $x + 3y + 10z = 24$, $2x + 17y + 4z = 24$.
- II Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ using LU decomposition method. Take $u_{11} = u_{22} = u_{33} = 1$.
- III Use Lagrange's formula to find the value of y when $x=10$, given:
- | | | | | |
|---|----|----|----|----|
| X | 5 | 6 | 9 | 11 |
| y | 12 | 13 | 14 | 16 |
- IV Solve $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ in the range $0 \leq x \leq 0.3$ using
 (i) Euler's method and (ii) modified Euler's method by choosing $h = 0.1$.
- V Derive the statement of second order and fourth order of Runge-Kutta method.

3. Answer in short (any five)

[5x5=25]

I Derive the false position method.

II Solve the system of equations $3x+y-z = 3$, $2x-8y+z = -5$, $x-2y+9z = 8$ using Gauss elimination method.

III Solve the system of equations $10x+y+z = 12$, $2x+10y+z = 13$ and $x+y+5z = 7$ by Gauss-Jordan method.

IV Find a polynomial satisfied by the following table:

X	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

V Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ by using Simpson's 1/3 rule.

VI Discuss the Simpson's 1/3 rule.

VII Solve $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0)=1$ in the range $0 \leq x \leq 0.2$ using Euler's method.

VIII Given $y' = x^2 - y$, $y(0) = 1$, find $y(0.1)$, $y(0.2)$ using Runge-Kutta second order method.

IX Interpolate by means of Gauss's backward interpolation formula, the sales of a concern for the year 1976, given that

year	1940	1950	1960	1970	1980	1990
sales	17	20	27	32	36	38

X Find y when $x = 9$, using Newton's forward formula of the tabulated below:

x	8	10	12	14	16
y	1000	1900	3250	5400	8950

Printing Pages : 2

Paper Code : MSc-Math-301 B (SVSU:2022-23/R)

Enrollment No.														
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Program Name-M.Sc
 III Semester /II Year Examination
 Subject Name: TOPOLOGY
 Subject Code: 301

[Time : 3:00 Hours]

[Max. Marks : 70]

Note : Attempt all the questions as per given instructions.

- I Objectives Question: Answer All Question [15×1=15]
- Every finite point set in a Hausdorff space S is
 (a) Closed (b) Open (c) Hausdorff (d) Connected
 - If X is a Hausdorff space, then a sequence of points of X
 (a) Converges to most one point of X
 (b) Converges to at least one point of X
 (c) Diverges to most one point of X
 (d) Diverges to at least one point of X
 - A set with same algebraic structure is called
 (a) Set (b) Space (c) Proper set (d) Topology
 - In a topological space (X, τ) , the members of τ are called
 (a) Open sets (b) Closed sets
 (c) Topology member (d) None of the above
 - In a topological space (X, τ) , the subclasses ϕ and X are
 (a) always open (b) always closed (c) connected (d) None of these
 - A is open if and only if $A \cap b(A) = \text{---}$?
 (a) ϕ (b) ϕ^c (c) X (d) A
 - Topology is derived from two greek words topos and logos, where the meaning of topos is—
 (a) Study (b) Geometry (c) Surface (d) None of these
 - If A and B are two subsets of a topological space, (X, τ) and they are disjoint. Then what will be $A \cap B$?
 (a) ϕ (b) $A \cup B$ (c) X (d) None of these
 - If $X = a, b, c$, then how many topologies are possible from set X .
 (a) 9 (b) 29 (c) 27 (d) 31
 - On a set X , which is weakest topology
 (a) Co-finite topology (b) Co-complement topology
 (c) Discrete topology (d) Indiscrete topology
 - For $n \geq 1$, R is
 (a) T_1 space (b) T_2 space (c) Hausdorff (d) None of these