19/05/2025

Printing Page: 2

Paper Code: MSc-Math-203 **A** **(SVSU:2024-25/R)**

Enrollment No.														

Programme: M.Sc. Math
Ist YEAR/II Semester EXAMINATION
Subject code: Msc-Math-203
Subject name: Mathematical Statistics

[Time : 3:00 Hours] **[Max. Marks : 70]**

I. Objectives Question: Answer All Question **[15×1=15]**

1 If two events A and B are independent, then $P(A \cap B) = ?$
A) $P(A) + P(B)$ B) $P(A) \times P(B)$ C) $P(A) / P(B)$ D) $P(A \mid B)$

2 Bayes' Theorem is used to find
A)Marginal probability B)Prior probability
C)Posterior probability D)Conditional variance

3 The probability that a point chosen at random from a line segment [0, 2] lies in [1, 2] is:
A) 1 B) 0.5 C) 0 D) 2

4 Which of the following is a valid property of the cumulative distribution function (CDF) F(x)?
A)F(x) is non-increasing B)F(x) ranges from $-\infty$ to ∞
C)F(x) is always 1 D)F(x)is non-decreasing

5 The Chebyshev's inequality provides a bound on:
A) The mean
B) The variance
C) The probability that a random variable deviates from its mean
D) The moment-generating function\

6 The moment-generating function (MGF) of a random variable X is defined as:
A)$E(X^r)$ B)$\sum XP(X)$ C)$E(e^{tX})$ D)$\sum e^{tX}P(x)$

7 Which of the following distributions is used to model the number of trials until the first success?
A) Binomial B) Poisson C) Geometric D) Normal

8 The variance of a Poisson distribution with mean λ is:
A) λ B) λ^2 C) $\sqrt{\lambda}$ D) $1/\lambda$

9 Which of the following is not a continuous distribution?
A) Normal B) Exponential C) Poisson D) Gamma

10 The Chi-square distribution is primarily used in:
A) Estimating population mean B) Variance testing
C) Regression analysis D) Mean testing

11 According to the Central Limit Theorem, the distribution of sample means tends to:
A) Chi-square B) t-distribution
C) Normal distribution D) Uniform distribution

12 Sampling distribution refers to:
A) The distribution of the population
B) The distribution of a sample
C) The distribution of a statistic based on a random sample
D) The uniform distribution

13 The value of the Pearson correlation coefficient lies between:
A) 0 and 1 B) −1 and 0 C) −1 and 1 D) 0 and ∞

14 In a simple linear regression, the slope represents:
A) Mean of Y B)Change in Y for a unit change in X
C) Error term D)Intercept

15 A Type I error occurs when:
A) A false null hypothesis is rejected
B) A true null hypothesis is rejected
C) A false nullhypothesis is accepted
D) A true alternative hypothesis is rejected

II. Long Question: Answer any Three **[10×3=30]**

1) Describe the procedure for testing a hypothesis.
Explain the concepts of null and alternative hypotheses, types of errors, level of significance, and p-value.
2) What is multiple correlation? Derive the formula for the multiple correlation coefficient between one variable and a group of two or more variables.
3) State and prove the Central Limit Theorem (CLT).Explain the practical implications of CLT in statistical inference.
4) Discuss the moment-generating function (MGF).
Derive the MGF of the normal distribution and show how it can be used to compute the mean and variance.
5)State and prove Bayes' Theorem. Explain its significance with a real-life example where posterior probability is calculated using prior and conditional probabilities.

III. Short Question: Answer any Five **[5×5=25]**

1) Define sample space and random experiment.
2) State and explain Chebyshev's inequality.
3) Find the mean and variance of a binomial distribution.
4) Differentiate between the normal and exponential distributions.
5) Define the moment-generating function (MGF).
6) Explain the Central Limit Theorem with an example.
7) Define sampling distribution.
8) Distinguish between correlation and regression.
9) What is rank correlation?
10) Explain the terms: Null hypothesis, alternative hypothesis, Type I and Type II errors.

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Printing Pages : 2

Paper Code: MSc-Math-204 **B** **(SVSU:2024-25/R)**

Enrollment No.													

M.Sc. (Mathematics)
IInd SEM EXAMINATION
Sub. Name –Operations Research
Sub.Code -M.Sc. Math-204

[Time : 3:00 Hours] **[Max. Marks : 70]**

I. Long Question: Answer Any Three **[10×3=30]**

1. Write a note on Operations Research and Define an Origin and Destination.
2. Explain in detail about assignment problem also differentiate with transportation problem.
3. Discuss the basic concept game theory with the help of suitable examples.
4. What is decision making? Give the important steps involved in decision making.
5. Explain Inventory with the help of some basic models.

II. Short Question: Answer any five **[5×5=25]**

1. Explain the Sensitivity analysis in terms of applications.
2. Discuss LIFO in Queuing theory.
3. Explain the Graphical method to solve a LPP with the help of one example.
4. Use simplex method to solve the following LPP- $3x_1 + 3x_2 \leq 6$, $x_1 + 4x_2 \leq 4, x_1, x_2 \geq 0$
5. Provide the difference between EOQ and EPQ inventory models.
6. Write the steps to solve a transportation problem with the help of suitable example.
7. Provide the advantages and disadvantages of Graphical method to solve a LPP.

III. Objectives Question: Answer All Question **[15×1= 15]**

1. The Two phase method is used to solve a
   (a)LPP (b)NLPP (c) Game theory problem (d) None of these
2. Replacement model can be written as
   (a)static models (b) dynamic models
   (c) both (a) and (b) (d) None of these
3. Allocation models are
   (a) iconic models (b) analogue models
   (c) symbolic models (d) None of these
4. The linear programming technique was developed during
   (a) world war –I (b) world war –II
   (c) 1930-35 (d) none

5. Graphical methods is the
(a) algebraic procedure for solving LPP
(b) geometric procedure for solving LPP
(c) both (a) and (b) (d) None of these

6. Sensitivity analysis
(a) is also called post -optimality analysis
(b) allows the decision maker to get more meaningful information
(c) provided the range within which a parameter may change without affecting optimality
(d) All of these

7. Transportation problem is special case of
(a)non-linear programming problem (b) left programming problem
(c) linear programming problem (d) None of these

8. Which of the following method generally gives better solution for T.P.
(a) Row minimum method (b) column minimum method
(c) Vogels method (d) None of these

9. Hungarian method is used for solving following problem
((a) non-linear programming problem (b) left programming problem
(c) linear programming problem (d) None of these

10. What aims at optimizing inventory levels
(a) inventory control (b) inventory capacity
(c) storage (d) None of these

11. What is concerned with the prediction of replacement cost and determination of the most economic replacement policy
(a) search theory (b) theory of replacement
(c) inventory contro l (d) None of these

12. In transportation models designed in linear programming, points of demand is classified as
(a)ordination (b)transportation (c) destinations (d) origins

13. What aims at optimizing inventory levels?
(a) Inventory Control (b) Inventory Capacity
(c) Inventory Planning (d) None of the above

14. Which of the following problem is a LPP?
(a)search theory (b)transportation
(c) inventory capacity (d) None of these

15. The operations Research technique, specially used to determine the optimum strategy is
(a) Decision Theory (b) Simulation
(c) Game Theory (d) None of the above

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Printing Pages : 2

Paper Code: MSc-Math-404 C (SVSU:2024-25/R)

Enrollment No.													

M.Sc-Mathematics

II YEAR IV SEMESTER EXAMINATION

Subject Name : Number Theory

Subject Code: Msc-Math-404

[Time : 3:00 Hours] [Max. Marks: 70]

I Objectives Question: Answer All Question [15×1=15]

1. The linear combination of gcd(252, 198) = 18 is?
   (a) 252*4 – 198*5 (b) 252*5 – 198*4
   (c) 252*5 – 198*2 (d) 252*4 – 198*4
2. The inverse of 3 modulo 7 is?
   (a) -1 (b) -2 (c) -3 (d) -4
3. The inverse of 7 modulo 26 is?
   (a) 12 (b) 14 (c) 15 (d) 20
4. The solution of the linear congruence 4x = 5(mod 9) is?
   (a) 6(mod 9) (b) 8(mod 9) (c) 9(mod 9) (d) 10(mod 9)
5. To one percent accuracy, the number of integers n in the list 04 , 1 4 , 2 4 , . . . , 10004 such that n % 16 = 1 is
   (a) 20 % (b) 50 % (c) 30 % (d) 35 %

6 The value of 5^{2003} mod 7 is?
(a) 3 (b) 4 (c) 8 (d) 9

7 The inverse of 19 modulo 141 is?
(a) 50 (b) 5 (c) 54 (d) 52

8 The linear combination of gcd(117, 213) = 3 can be written as ________
(a) 11*213 + (-20)*117 (b) 10*213 + (-20)*117
(c) 11*117 + (-20)*213 (d) 20*213 + (-25)*117

9 Both addition and multiplication in z are......................
(a) commutative (b) Associative
(c) Distributive (d) all of these

10 If a=qb for some inters q and $a, b \neq 0$ then we say that
(a) b divides a (b) b is a factor of a
(c) a is a multiple of b (d) all of these

11 Let n(A) denotes the number of elements in set A. if n(A) = p and n(B) = q, then how many ordered pairs (a,b) are there with $a \in A$and $b \in B$?
(a) p^2 (b)p x q (c) p + q (d) 2 pq

12 If A be a finite set of size n, then number of element in the power set of AxA
(a) 2^{2n} (b)2^{n^2} (c) $(2n)^2$ (d) none of these

13 $A-(B\cup C)$ is

$(a)(A-B)\cup(A-C)$ (b) $A-B-C$

$(c)(A-B)\cap(A-C)$ (d) $A-(B\cap C)'$

14 The number of elements in the power set of the set {{a, b}, c} is

(a). 8 (b). 4 (c). 3 (d). 7

15 In a language survey of students it is found that 80 students know English, 60 know French, 50 know German, 30 known English and French, 20 know French and German, 15 know English and German and 10 students know all the three languages. How many students know at least one language?

(a.) 135 (b). 30 (c.) 10 (d) 45

II. Long Question: Answer any Three **[10×3=30]**

1. Apply Wilson's theorem to show that $18!+1\equiv 0(\text{mod } 19)$ and $18!+1\equiv 0(\text{mod } 23)$.
2. Prove that the number of primes is infinite.
3. State and prove the Chinese Remainder theorem.
4. Use Euclidean algorithm to obtain the g.c.d. 'd' of 3997 and 2947. Also find integers x and y such that $d=3997x+2947y$
5. If p and q are distinct odd primes, then show that $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right)=(-1)^{\left(\frac{p-1}{2}\right)\left(\frac{p-1}{2}\right)}$

III. Short Question: Answer any Five **[5×5=25]**

1. If a, b are integers such that $(a,b)=1$ then show that $(a+b,a-b)=1$
2. Find the values of $\left(\frac{2}{17}\right)$ and $\left(\frac{-2}{17}\right)$.
3. Find two Pythagorean triples whose terms form an arithmetic progression.
4. Find highest power of 5 dividing 3000!
5. Find the last digit in the ordinary decimal representation of 3^{101}.
6. Define perfect number show that an even perfect number n ends in digit 6 or 8
7. Find the remainder when 15 is divided by 17.
8. Define the Mobius function.
9. If gcd (a, b) = 2 then find gcd (a, b + 3a).
10. If x and y are odd integers prove that x^2+Y^2 is even but not divisible by 4.

19/05/2025

Printing Pages : 2

Paper Code: MSc-Math-407 B (SVSU:2024-25/R)

Enrollment No.													

Program Name- M.Sc. Mathematics

IV Semester / II Year Examination

Subject Code: MSc-Math-407

Subject Name: Fuzzy Sets and Its Applications

[Time : 03:00 Hours] [Max. Marks : 70]

Note : 1. Attempt all the questions as per given instructions.

I. Objectives Question: Answer All Question [15×1=15]

1. The Principle of uncertainty invariance was introduced by

(a) Renyi (b) Jumarie (c) Wonneberger (d) Klir

2. For a modifier h, which is not true

(a) h(0) = 0, h(1) = 1 (b) h is continuous
(c) h is strong (d) none

3. Quantifiers of first kind also known as

(a) relative quantifiers (b) absolute quantifiers
(c) linguistic hedge (d) none

4. Extension principle is a fuzzification of

(a) fuzzy sets (b) fuzzy function (c) crisp function (d) none of these

5. Interval valued fuzzy sets introduced by

(a) Zadeh (b) Cauchy (c) Prade (d) Lagrange

6. The support of a fuzzy set is a

(a) crisp set (b) fuzzy set (c) a number (d) none of these

7. The height of a fuzzy set is a

(a) crisp set (b) fuzzy set (c) a number (d) none of these

8. The α-cut set of any set is a

(a) crisp set (b) fuzzy set (c) a number (d) none

9. The α-cut set is also called

(a) α-level set (b) cut worthy set (c) both A and B (d) none

10. The concept of fuzzy measure was introduced by

(a) Zadeh (b) Wang and Klir (c) Sugeno (d) Glen Shafer

11. Fuzzy set was introduced by

(a) Zadeh (1965) (b) Lowen (1967)
(c) Gottwald (1971) (d) Kosko (1977)

12. The concept of L-fuzzy sets was introduced by

(a) Goguen (1965) (b) Goguen (1967)
(c) Zadeh (1965) (d) Zadeh (1973)

13. A representation of fuzzy sets in terms of their α-cuts was introduced by

(a) Zadeh (b) Yager (c) Chang (d) Wong

14. The most commonly used range of values of membership functions is

(a) [0, 1[(b)]0, 1] (c)]0, 1[(d) [0, 1]

15. The characteristic function of a crisp set may assigns a value

(a) 1 (b) 0 (c) either 1 or 0 (d) none of these

II. Long Question : Answer Any Three **[10×3=30]**

1. Explain why we need fuzzy set theory? Also give some examples of prospective fuzzy variables in daily life.
2. State and prove first decomposition theorem on fuzzy sets.
3. State and prove second characterization theorem on fuzzy compliments.
4. Let $\tilde{A}$ be a fuzzy set defined by $\tilde{A}=\left\{\frac{0.5}{x_1},\frac{0.4}{x_2},\frac{0.7}{x_3},\frac{0.8}{x_4},\frac{1}{x_5}\right\}$.

   Find all α cut set and strong α cut sets.

III. Short Question: Answer any Five **[5×5=25]**

1. Define fuzzy number. Write down the conditions for a fuzzy set to be a fuzzy number.
2. Define α-cut, strong α-cut sets of a given fuzzy set.
3. Show that a fuzzy set $\tilde{A}$ on R is convex iff
   $\tilde{A}(\lambda x_1+(1-\lambda)x_2)\geq\min[\tilde{A}(x_1),\tilde{A}(x_2)]\,\forall x_1,x_2\in R$ and all $\lambda\in[0,1]$.
4. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets defined on a universal set X. Then show that $|\tilde{A}|+|\tilde{B}|=|\tilde{A}\cup\tilde{B}|+|\tilde{A}\cap\tilde{B}|$.
5. State second decomposition theorem.
6. Show that every fuzzy compliment has at most one equilibrium.
7. Show that if $\tilde{R}$ is symmetric then each power of $\tilde{R}$ is symmetric.

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Printing Pages : 2

Paper Code: MSc-Math-404 C (SVSU:2024-25/R)

Enrollment No.													

M.Sc-Mathematics

II YEAR IV SEMESTER EXAMINATION

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(a) $(A - B) \cup (A - C)$ (b) $A - B - C$

(c) $(A - B) \cap (A - C)$ (d) $A - (B \cap C)'$

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III. Short Question: Answer any Five **[5×5=25]**

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Printing Pages : 2

Paper Code: MSc-Math-407 **B** **(SVSU:2024-25/R)**

Enrollment No.														

Program Name- M.Sc. Mathematics
IV Semester / II Year Examination
Subject Code: MSc-Math-407
Subject Name: Fuzzy Sets and Its Applications

[Time : 03:00 Hours] **[Max. Marks : 70]**

Note : 1. Attempt all the questions as per given instructions.

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7. The height of a fuzzy set is a
(a) crisp set (b) fuzzy set (c) a number (d) none of these

8. The α-cut set of any set is a
(a) crisp set (b) fuzzy set (c) a number (d) none

9. The α-cut set is also called
(a) α-level set (b) cut worthy set (c) both A and B (d) none

10. The concept of fuzzy measure was introduced by
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(a) [0, 1[(b)]0, 1] (c)]0, 1[(d) [0, 1]

15. The characteristic function of a crisp set may assigns a value

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II. Long Question : Answer Any Three **[10×3=30]**

1. Explain why we need fuzzy set theory? Also give some examples of prospective fuzzy variables in daily life.
2. State and prove first decomposition theorem on fuzzy sets.
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4. Let $\tilde{A}$ be a fuzzy set defined by $\tilde{A} = \left\{ \frac{0.5}{x_1}, \frac{0.4}{x_2}, \frac{0.7}{x_3}, \frac{0.8}{x_4}, \frac{1}{x_5} \right\}$.

   Find all α cut set and strong α cut sets.

III. Short Question: Answer any Five **[5×5=25]**

1. Define fuzzy number. Write down the conditions for a fuzzy set to be a fuzzy number.
2. Define α-cut, strong α-cut sets of a given fuzzy set.
3. Show that a fuzzy set $\tilde{A}$ on R is convex iff
   $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min[\tilde{A}(x_1), \tilde{A}(x_2)] \, \forall x_1, x_2 \in R$ and all $\lambda \in [0,1]$.
4. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets defined on a universal set X. Then show that $|\tilde{A}| + |\tilde{B}| = |\tilde{A} \cup \tilde{B}| + |\tilde{A} \cap \tilde{B}|$.
5. State second decomposition theorem.
6. Show that every fuzzy compliment has at most one equilibrium.
7. Show that if $\tilde{R}$ is symmetric then each power of $\tilde{R}$ is symmetric.

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Paper Code: MSc-Math-202 A (SVSU:2024-25/R)

Enrollment No.													

M.Sc.-Mathematics

I -YEAR IInd Semester EXAMINATION

Complex Analysis (MSc-Math-202)

[Time : 03:00 Hours] [Max. Marks : 70]

Note: Attempt all the questions as per given instructions.

1. Multiple Choice Questions (MCQs) [01×15=15]

I Every uniformly convergent series is
(a) not convergent (b) convergent (c) Not absolutely (d) none.

II Radius of convergent of Analytic function in its domain is
(a) $|\int_L f(z)\, dz| = \int_L |f(z)|\, |dz|$ (b) 1 (c) 0 (d) ∞.

III The Radius of convergence of the series of sinz is:
(a) 0 (b) 1 (c) -1 (d) none

IV The Polar form of complex number : $-5+5i$ is
(a) $5\sqrt{2}\, e^{-i\pi/4}$ (b) $5\sqrt{2}\, e^{-i3\pi/4}$ (c) $5\sqrt{2}\, e^{i3\pi/4}$ (d) none.

V The value of $\frac{1}{2\pi i}\int_{|z|=4}\frac{e^{bz}}{z-3}\, dz$ is :
(a) 0 (b) e^{3b} (c) e^{2b} (d) e^{-2b}

VI The value of exp(z + 2πi) is :
(a) exp(z) (b) exp(z) + z
(c) exp (z) + z +1 (d) exp(z) - z -1.

VII The value of $\frac{1}{2\pi i}\int_{|z|=3}\frac{e^{z}}{z+2}\, dz$ is :
(a) 0 (b) e^3 (c) e^{-2} (d) 1.

VIII Residue of $z^2/(z-1)(z-2)(z-3)$ at z =1 is:
(a) - 4 (b) 1/2 (c) -1 (d) none

IX If, $\int_c \frac{dz}{(z-a)^n}$, where n = 2,3,4, ,... ... where z = a is inside the simple closed curve C has the value
(a) πi (b) $2\pi i$ (c) $-\pi i$ (d) 0.

X The value of $\frac{1}{2\pi i}\int_{|z|=4}\frac{e^{z}}{(z+2)^2}\, dz$ is :
(a) 0 (b) e^3 (c) e^{-2} (d) 1.

XI Find the fixed point of the bilinear transform ,
$w = \frac{z}{z-2}$ are
(a) 0 and 3 (b) 0 and 2 (c) 1 and 3 (d) 0 and 1

XII The value of $\Gamma\,\frac{1}{2}$ is

(a) $\sqrt{2\pi}$ (b) $\sqrt{\pi}/2$ (c) $\sqrt{\pi}$ (d) none of these.

XIII The value of $\Gamma\,1$ is

(a) 0 (b) 1 (c) $\sqrt{\pi}$ (d) none of these.

XIV The value of $\Gamma\,2$ is :

(a) 0 (b) 1 (c) $\sqrt{\pi}$ (d) none of these.

XV The value of $\Gamma z\ \Gamma(1-z)$ is

(a) $-\frac{\pi}{\sin(\pi z)}$ (b) $\frac{\pi}{\sin(\pi z)}$ (c) $-\pi i$ (d) none .

2. Answer in long (any three) **[10×3=30]**

I State and prove Cauchy's integral formula for first derivative of $f(z)$.

II The composition of two bilinear transformations is a bilinear transformation, Prove or Disprove.

III Evaluate the Residue of $z^3/(z-1)^4(z-2)(z-3)$ at $z=1, 2, 3$.

IV State and prove the Poission's Integral formula.

V State and prove Jensen's theorem.

3. Answer in short (any five) **[5x5=25]**

I State and prove the Cauchy Integral Formula.

II Expand $\log\left(\frac{1+z}{1-z}\right)$ at $z=0$, using Taylor series.

III Consider the map $w=\frac{1}{z}$ and determine the region R' in w-plane of the infinite strip R bounded by $\frac{1}{4}<y<\frac{1}{2}$.

IV Find the bilinear transformation which maps the points $z_1=2, z_2=i, z_3=-2$ into the points $w_1=1, w_2=i, w_3=-1$.

V State and prove maximum modulus theorem.

VI Write down the statement of Borel's Theorem.

VII Find the value of : $\frac{1}{2\pi i}\int_c^{\square} \frac{e^{2z}}{(z-a)^2}\ dz$

VIII Expand $\frac{1}{z^2-3z+2}$ in the region (i) $|z|<1$ (ii) $|z|>2$ (iii) $1<|z|<2$.

IX Define conformal mapping with one example.

X Write down the statement of the Hadmard's Factorization theorem.

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Printing Pages :2

Paper Code: M.Sc-Math-401 A (SVSU:2024-25/R)

Enrollment No.													

Program Name: MSc-MATHEMATICS

IV- Semester /II Year Examination

Subject Code: MSc-MATH-401

Subject Name: FUNCTIONAL ANALYSIS

[Time : 03:00 Hours] [Max. Marks : 70]

Note : 1. Attempt all the questions as per given instructions.

1. Multiple Choice Questions (MCQs) **[01×15=15]**

I The linear span of empty set equals:
(a) Empty set (b) Zero subspace (c) The whole space (d) None of these

II Which of the following is not a linear space over Q ?
(a) 2Q (b) Z (c) Q (d) None of these

III Which of the following is not a linear space ?
(a) C over R (b) Q over R (c) R over Q (d) C over Q

IV Dimension of R^n as a linear space over C is :
(a) n (b) n + 1 (c) n^2 (d) 2n

V Dimension of C^n as a linear space over R is :
(a) n (b) n + 1 (c) 2(n + 1) (d) 2n

VI C over R is a
(a) Field (b) Vector space (c) Banach Space (d) all of these

VII If E is finite dimensional linear space of dimension n, and F is a subset of E with m elements, where m < n, then which of the following is true?
(a) F can span E (b) F is linearly independent in E
(c) F is linearly dependent in E (d) F cannot be a basis of E

VIII If X and Y are normed spaces, and if T : X →Y is a linear operator, then T is bounded if and only if:
(a) T maps bounded subsets of X into bounded subsets of Y,
(b) T maps open subsets of X into open subsets of Y,
(c) both (a)&(b)
(d) none.

IX If X, Y are normed spaces and if A : X →Y is a bijective, bounded linear map, then:
(a) A is always an open map,
(b) A is an open map if X is a Banach space.
(c) A is an open map if Y is a Banach space.
(d) A is an open map if both X and Y are Banach spaces.

X Which of the following is true?
(a) If A,B are invertible linear operators on X, then A+B is invertible,
(b) If A,B are invertible linear operators on X, then A−B is invertible,
(c) If A,B are invertible linear operators on X, then AB is invertible,
(d) If A is invertible linear operator on X, and k is any scalar, then kA is invertible.

XI If X and Y are normed spaces, then the space of bounded linear operators L(X, Y) is a Banach space if and only if:
(a) X is a Banach's space,
(b) Y is a Banach's space,
(c) Both X and Y are Banach's spaces,
(d) Both X and Y are finite dimensional spaces.

XII For any bounded linear operator A : X → Y, kerA is:
(a) a closed subspace of Y, (b) an open subspace of Y,
(c) a closed subspace of X, (d) an open subspace of X.

XIII For any normed space X, the dual space X^* is:
(a) (a) Always a Banach's space, (b) Always a compact set,
(c) Always finite dimensional, (d) Always an infinite dimensional

XIV Dimension of C^n as a linear space over C is :
(a) 2n (b) n (c) n+1 (d) n+2

XV Any bounded subset of R is
(a) compact, (b) connected, (c) relatively compact, (d) none

2. **Answer in long (any three)** **[10×3=30]**

I If L is an inner product space. Show that $\sqrt{(x,x)}$ has the properties of a norm.

II State and prove Bessel's inequality.

III Let T be an operator on a Hilbert space H. Then there exists a unique operator T* on H such $(T_x, y) = (x, T_y^*)$ for all $x, y \in H$. The operator* is called the adjoint of the operator T.

IV State and prove the closed graph theorem.

V State and prove Hahn Banach Theorem.

3. **Answer in short (any five)** **[5x5=25]**

I Prove that in a normed linear space, every convergent sequence is a cauchy sequence.

II Let N and N^1 be normed liner space and let $T: A \to N^1$ be any linear transformation. If N is finite dimensional, then prove that T is continuous (or bounded).

III If A_1 and A_2 are self-adjoint operators on H, then their product A_1A_2 is self-adjoint ⇔ $A_1A_2=A_2 A_1$

IV An operator T on a Hilbert space H is self-adjoint ⇔ (T_x, x) is real for all x.

V Define ortho normal sets with example.

VI Show that in a Hilbert space H an ortho-normal sets is complete if any only if $x \perp s \Rightarrow x = 0$.

VII If N is a normal operator on a Hilbert space H, Then prove that $\|N^2\| = \|N\|^2$.

09/05/2025

Printing Pages :2

Paper Code: M.Sc-Math-201 B (SVSU:2024-25/R)

Enrollment No.													

Program Name M.Sc. (Mathematics)
II Semester / ...I Year Examination
Subject Code M.Sc. Math 201
Subject Name - Metric Space

[Time : 3:00 Hours] **[Max. Marks : 70]**

I Objectives Question: Answer All Question **[15×1=15]**

1. A metric space is compact if and only if it is
(a) countably not compact (b) uncountably not compact
(c) countably compact (d) uncountably compact

2. If a Cauchy sequence has a convergent subsequence then it is
(a) divergent (b) convergent (c) both a and b (d) none of these

3. In interval $[a, b]$ usual metric space is
(a) open set (b) closed set (c) Half open set (d) Half closed set

4. The metric space [0,1] with absolute value metric space is
(a)not bounded (b) totally bounded
(c) complete (d) both b and c

5. If A is an open set in metric space (M, ρ) then A^c in (M, ρ) is a
(a) closed set (b) open set (c) both a and b (d) none of these

6 The set of real numbers is
(a) a complete ordered field (b) an incomplete ordered field
{c}A and B both true (d)all are true

7 A metric space is disconnected if and only if it is union of two
(a)non-empty sets (b)disjoint sets
(c)may be connected (d)none of these

8 If (X,d) be a pseudo-metric space, then
(a)d(x,y)=0 if x=y (b)d(x,y)=0 iff x=y
(c)A and B both are true (d)none of these

9 If (X,d) be a metric space and for x,y ϵ X, then
(a)d(x,y) is positive (b)d(x,y) is negative
(c)A and B both are true (d)none of these

10 If (X,d) be a metric space and for x,y ϵ X, then
(a) d(x,y)=0 if x=y (b) d(x,y)=0 iff x=y
(c) A and B both are true (d) none of these

11 If (X,d) be a metric space and for x,y,z ϵ X, then d(x,y) is
(a)< d(x,z)+ d(z,y) (b)$\leq$ d(x,z)- d(z,y)
(c)$\leq$ d(x,z)+ d(z,y) (d)none of these

12 Example of pseudo-metric space is
(a) $|x^2 - y^2|$ (b) $|x^3+y^3|$ (c) $|x+y|$ (d) $|x-y|$

13 If (X,d) be a metric space and $d_1(x,y)= 2d(x,y)$ then
(a) d_1 is not a metric (b) d_1 is also a metric
(c) A and B both are true (d) none of these

14 The distance between a set A and an empty set φ is
(a) 1 (b) 0 (c) infinite (d) none of these

15 Every metric space is pseudo metric space
(a) TRUE (b) FALSE
(c) A and B both are true (d) none of these

II. Long Question: Answer any Three **[10×3=30]**

1. A subset in metric space is open if and only if it is a neighborhood of each of its points
2. Describe Makowski's inequality
3. Show that every compact metric is complete.
4. Let (X_1,d_1) and (X_2,d_2) be any two metric spaces Define d by $d=\sqrt{[(d_1^2(x_1,y_1) + d_2^2(x_2,y_2))]}$ where $x_1,y_1 \in X_1$ *and* $x_2,y_2 \in X_2$ *prove* that d is metric for $X_1 x X_2$
5. Show that every metric space is first countable.

III. Short Question: Answer any Five **[5×5=25]**

1. Define complete metric space.
2. Define Interior and exterior point of a set.
3. Show that closed subset of compact sets are compact.
4. Define Cauchy sequences in a metric space.
5. A subset of metric space X is closed if and only if $D(A) \subset A$ i.e iff A contains all it limit points.
6. Show that in a metric space, every closed sphere is a closed set.
7. In a metric space the intersection of a finite number of open sets is open
8. For a usual metric $d(x,y) =|x - y|$ for $[0, 1]$ describe $S(1/4,1/4)$,$S[1/4, 1/4]$
9. Define first and second countable Spaces.
10. Define Separated sets of a metric space.