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Paper Code: BPCM-103

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(SVSU:2024-25/S)

Enrollment No.

B.Sc. (PCM) BPCM-103
I YEAR/I SEM EXAMINATION
Calculus - I

[Time : 3:00 Hours]

[Max. Marks : 100]

I Long Question : Answer Any Three

[15×3=45]

1. Prove that the function $f(x) = |x|$ is continuous at $x = 0$ but not differentiable at $x = 0$.

2. $y = e^{\tan^{-1}x}$ prove that

$$(1+x)^2 y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$$

3. Find the all asymptotes of the curve

$$3x^2 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7x^2 + 4x + 5y = 0$$

4. Show that the pedal equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2 + b^2}$$

5. Expand $\log(1+x)$ by McLaurin theorem

II. Short Question: Answer any Five

[5×5=25]

1. Find the right hand and left hand limits of $\lim_{x \rightarrow 2} \frac{2x^2 - 8}{x - 2}$

2. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\frac{y}{x}$ show that $x \frac{du}{dx} + y \frac{du}{dy} = 0$

3. Find the equation of the tangent at the point t to the cycloid
 $x = a(t + \sin t)$ $y = a(1 - \cos t)$

4. Show that curve $r = a(1 + \sin \theta)$ and $r = a(1 - \sin \theta)$ cut orthogonally.

5. Find the radius of curvature at the point (x, y) on the curve
 $y^2 = 4ax$.

6. Find the asymptotes of the curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

7. Find the c of the mean value theorem is

$$f(x) = x(x-1)(x-2); a = 0, b = 1/2$$

8. Write the statement of ROLLE'S Theorem.

9. State and Prove Euler's theorem.

10. If $u = \log \frac{x^4 + y^4}{x + y}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$

III. Objectives Question: Answer All Question

[15×2=30]

- $\lim_{x \rightarrow 0} \frac{|x|}{x}$ is equal to :
(a) 1 (b) -1 (c) 2 (d) does not exist
- $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$ is equal to :
(a) 0 (b) 1 (c) -1 (d) 2
- A function $f(x)$ is differentiable at $x = a$ if
(a) $Rf'(a) = Lf'(a)$ (b) $Rf'(a) = 0$
(c) $Lf'(a) = 0$ (d) $Rf'(a) \neq lf'(a)$
- The function $f(x) = |x - 1|$ not differentiable at
(a) $x = 0$ (b) $x = -1$ (c) $x = 1$ (d) $x = 2$
- By Leibnitz's theorem find the n^{th} differential of the of two function
(a) sum (b) difference (c) product (d) quotient
- $y = \sin(ax + b)$ then $D^n \sin(ax + b)$
(a) $a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$ (b) $a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$
(c) $a^n \tan\left(ax + b + \frac{n\pi}{2}\right)$ (d) none of these
- If u is a homogeneous function of x and y of degree n then
 $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to:
(a) n (b) nu (c) $\frac{n}{u}$ (d) u
- By Tayler's theorem second term in the expansion of $\log \sin x$ power of
 $(x - a)$ is
(a) $(x - a) \cot a$ (b) $(x - a) \operatorname{cosec} a$
(c) $(x - a) \cot a \operatorname{cosec} a$ (d) $(x - a)^2 \cot a$
- For the cardioid $r = a(1 - \cos \theta)$ the value of ϕ is
(a) θ (b) $\theta/2$ (c) $-\theta/2$ (d) $-\theta$
- Two curve cut orthogonally if $\tan \phi_1 \tan \phi_2$ is equal to
(a) 1 (b) 0 (c) -1 (d) none of these
- Pedal formula for radius of curvature is
(a) $\frac{1}{r} \frac{dr}{d\theta}$ (b) $\frac{1}{r} \frac{dr}{dp}$ (c) $\frac{1}{r} \frac{dp}{dr}$ (d) $r \frac{dp}{dr}$
- The number of asymptotes of the curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ is
(a) 2 (b) 3 (c) 4 (d) 1
- The asymptotes of the curve $y^2(x^2 - a^2) = x$ which are parcellel to x -axis are :
(a) $x = \pm a$ (b) $y = \pm a$ (c) $y = 0$ (d) $x = 0$
- If $x = r \cos \theta$ $y = r \sin \theta$ then $\frac{\partial y}{\partial x}$ is
(a) $\tan \theta$ (b) $-\cot \theta$ (c) $\cos \theta$ (d) none of these
- If $u = f(y/x)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$:
(a) 0 (b) 1 (c) 2 (d) -1