22|12|2025

Printing Pages: 2

Paper Code: B030501T B (SVSU:2025-26/R)

Enrollment No.													

Program Name: B.Sc. Mathematics/Physics/CS/IT

V-Semester / III-Year Examination

Subject Code: B030501T

Subject Name: Groups, Ring and Linear Algebra

[Time: 03:00 Hours] **[Max. Marks: 75]**

Note: 1. Attempt all the questions as per given instructions.

1. Multiple Choice Questions (MCQs) **[01×15=15]**

I The group of automorphisms of an infinite cyclic group is of order
(a) 0 (b) 1 (c) 2 (d) 5

II If F is a field and f(x), g(x) are two non zero elements of F[x], then deg[f(x), g(x)]=
degf(x) + degg(x) (b) degf(x) - degg(x)
(c) degf(x).degg(x) (d) none of these

III Let $f(x) = 2 + 6x + 4x^2$, $g(x) = 2x + 4x^2$ be two polynomials over the ring $(I_8, +_8, x_8)$ then deg (f(x) + g(x)) is
(a) 0 (b) 1 (c) 2 (d) 4

IV Which of the following is true?
(a) A field has proper ideals.
(b) Every Euclidean has unity element.
(c) Every Integral domain is a field.
(d) Ring of integers is a field.

V The set Z of self-conjugate elements ofa group G is called the .... of G.
(a) Normalizer (b) centre (c) kernel (d) subgroup

VI The zero vector in the vector space R^4 is
(a) 0, (b) (0, 0) (c) (0,0,0,0) (d) none of these

VII Let V be a vector space over the field of F. The intersection of any collection of subspaces of V is
(a) Zero subspace of V (b) trivial subspace of V
(b) (c) a subspace of V (d) none of these

VIII A set of vectors which contains the zero vector is
(a) Linearly independent (b) Linearly dependent
(c) both (d) none of these

IX The zero subspace have the dimension
(a) 1 (b) 2 (c) zero (d) 3

X If W be a subspace of a finite dimensional vector space V(F), then dim V/W
(a) dim V -dim W (b) dim V -dim W
(b) (c) dim V/dim W (d) none of these

XI Two finite dimensional vector spaces over the same field are isomorphic iff they are of same dimension. It is known that
(a) Exisence theorem for vector space
(b) Extension theorem for vector space
(c) Dimension theorem for vector space
(d) Isomorphism theorem for vector space

XII Let U be an n dimensional vector space over the field F,and let V be an m dimensional vector space over F. Then the vector space L(U, V) of all linear transformation from U into V is finite dimensional and is of dimension

(a) M (b) n (c) mn (d) none of these

XIII Similar matrices have ....determinant.

(a) Equal (b) opposite (c) unequal (d) inverse

XIV Let V be an n-dimensional vector space over the field F. the dimension of the dual space of V is

(a) N (b) 2n (c) n/2 (d) none of these

XV If S is any subset of a vector space V(F), then S^0 is a subspace of

(a) V (b) V' (c) V'' (d) none of these

2. Answer in long (any three) **[10×3=30]**

I The union of two subspaces of a vector space is not necessarily is a subspace, also give an example in support.

II Let U and V be vector spaces over the field F and let T be a linear transformation from U into V. Suppose U is finite dimensional, then rank(T)+nullity(T) = dim.(U).

III Show that the vectors (1, 1, -1), (2, -3, 5) and (-2, 1, 4) of R^3 are linearly independent.

IV If $\alpha_1, \alpha_2, \ldots, \alpha_n \in V$ are linearly independent then every element in their linear span has a unique representation in the form $a_1\alpha_1+a_2\alpha_2+\ldots+a_n\alpha_n$ with the $a_i \in F$.

V Show that the set of all automorphisms of a group forms a group with respect to composite of functions as the composition.

3. Answer in short (any five) **[6x5=30]**

I Clearly explain the range and null space of a linear transformation.

II Prove that every homomorphic image of a commutative ring is a commutative ring.

III Discuss the vector space along with conditions.

IV Show that the set R{x} of all polynomials over an arbitrary ring R is a ring with respect to addition and multiplcation of polnomilas.

V Show that every finite integral domain is a field.

VI Explain the linear transformation along with example.

VII Derive the ring with properties.

VIII Show that the vectors (1, 1, -1), (2, -3, 5) and (-2, 1, 4) of R^3 are linearly independent.

IX The intersection of any two subspaces of a vector space is a subspace.

X Let F be the field of complex numbers and let T be the function from R^3 onto R^3 defined by
$T(a_1, a_2, a_3) = (a_1 - a_2+2a_3, 2a_1+a_2 - a_3, - a_1 - 2a_2)$. Verity that T is a linear transformation. Describe the null space of T.

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Printing Pages :2

Paper Code: BSC-H-Math-507 A SVSU:2023-24/R)

Enrollment No.													

BSC-H-MATH-507

IIIrd -YEAR Vth SEMESTER EXAMINATION

Discrete Mathematics

[Time : 3:00 Hours] [Max. Marks : 100]

I. Objectives Question: Answer All Question [15×2=30]

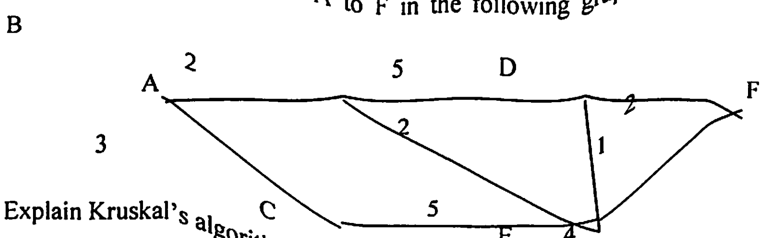
1. In Boolean Algebra ,a + a is equal to :
   (i) 2a (ii) a (iii) 1 (iv) 0
2. In Boolean Algebra ,a + a' is equal to :
   (i) 2a (ii) a (iii) 1 (iv) 0
3. In Boolean Algebra ,a + (a.b) is equal to:
   (i) b (ii) a (iii) 1 (iv) 0
4. The output of a 2 input OR gate is 0 only when its
   (i) both input are 0 (ii) either input is 0
   (iii) both input are 1 (iv) either input is 1
5. In Boolean Algebra , a.(a + b. (a + b)) is equal to :
   (i) a (ii) c (iii) a + b (iv) c.b
6. The sum of products form of ab + bc' is:
   (i) abc + abc' +a' bc' (ii)abc + abc' +a' b'c'
   (iii) abc' + a' bc' +a' b'c' (iv) abc' + a'bc' +a' b'c
7. The maximum number of edges in a simple graph with n vertices is :
   (i) $\frac{n(n-1)}{2}$ (ii) $\frac{n(n+1)}{2}$ (iii) $\frac{(n-1)}{2}$ (iv) $\frac{n(n-1)}{3}$
8. The minimum number of edges in a connected graph with n vertices is:
   (i) n− 1 (ii) n + 1 (iii) n (iv) none of these
9. The list of all of the minterms of the two variables x and y are
   (i)xy,x'y ,xy',x'y' (ii) xy,x' ,xy',x'y'
   (iii)xy,x'y ,y',x'y' (iv) xy,x'y ,xy',x'
10. In a bounded distributive lattice if a complement exists it is :
    (i)unique (ii) different (iii) both (i) and (ii) (iv) none
11. Every distributive lattice is:
    (i) Modular (ii) not modular (iii) both (i) and (ii) (iv) none
12. There is no horizontal line in a Hasse diagram of a poset
    (i)False (ii) True (iii) both (i) and (ii) (iv) none
13. The list of all of the maxterms of the two variables x and y are
    (i) x +y,x' +y ,x +y',x'+ y' (ii) x +y,x' ,x+y',x+'y'
    (iii)xy,x'+y ,y',x+'y' (iv) x+y,x+'y ,x+y',x'

14. In Boolean Algebra ,a .(a + b) is equal to:
(i) b (ii) a (iii) 1 (iv) 0

15. In Boolean Algebra ,aa' is equal to
(i) b (ii) a (iii) 1 (iv) 0

II Long Question : Answer Any Three **[15×3=45]**

1. Make Hasse diagram for $\{(a.b):a\leq b\}$ on the set $\{1,2,3,4\}$.
2. (i) In Boolean Algebra if $a + b = 1$ and $a.b= 0$ than show that $b = a'$
   (ii) Use Karnaugh map to simplify the : $X = ABC' + ABC$.
3. If (L, Λ, V) be a complemented and distributive lattice, then for any $a, b \in L$, prove that:
   (i) $\overline{aVb} = \overline{a} \Lambda \overline{b}$ (ii) $\overline{a\Lambda b} = \overline{a} V \overline{b}$.
4. Find the shortest path from A to F in the following graph by Dijekstra's Algorithm.



5. Explain Kruskal's algorithm on tree with an example.

III. Short Question: Answer any Five **[5×5=25]**

1. Write down the De'Morgan's Law for set theory.
2. Define Hasse diagram with an example.
3. Define Lattices with example.
4. In Boolean Algebra , show that:
   $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(a + b')(c' + a)$
5. Define derived connectives NAND,NOR and XOR gates with the help of truth table.
6. Define degree of a vertex.
7. Prove that a graph G with n vertices is a tree if and only if it has n-1 edges and no cycles.
8. When in a graph said to be a Hamiltonian graph?

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Printing Pages : 2

Paper Code :BSc-H-Math-502 C (SVSU:2022-23/R)

Enrollment No.												

B.Sc.-H-MATH

V-Semester / III- Year Examination

Subject Code B.Sc.-H-MATH -502

Subject Name: GUROP THEORY- II

[Time : 3:00 Hours] [Max. Marks : 100]

I Long Question : Answer Any Three [15×3=45]

1. Show that the set of all automorphisms of a group forms a group with respect to composite of functions as the composition.
2. State and Prove Fundamental theorem of finite abelian groups.
3. If G_1 and G_2 are groups than (i) $G_1 \times \{e_1\} \cap \{e_1\} \times G_1 = \{(e_1, e_2)\}$ ie the identity is the only element common to $G_1 \times \{e_1\}$ and $\{e_1\} \times G_2$ (ii) Every element of $G_1 \times \{e_2\}$ commutes with every element of $\{e_1\} \times G_2$.
4. If H is a p-sylow subgroup of G and $x \in G$ then $x^{-1}Hx$ is also a p-Sylow subgroup of G.
5. State and Prove Sylow's Thoerem

II. Short Question: Answer any Five [5×5=25]

1. Prove that if p is prime number, then any group G of order 2p has a normal subgroup of order p.
2. Prove that p-Sylow sub-group of a group G of order 33 is a normal subgroup of G
3. A group G is the direct product of its two sub groups H and K if and only if Hand K are are normal subgroups of G.
4. Prove that Cauchy's theorem for abelian group.
5. Show that the symmetric group P_n is not solvable for n is greater and equal to five.
6. Give the statement of first Sylow Theorem.
7. Explain Stabilizers and kernel of group action .
8. Prove that $f : a \rightarrow a^{-1}$ is an automorphism of group G iff G is abelian.
9. Prove that Klein four group K_4 is cyclic.
10. Define isomorphic mapping.

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Printing Pages :2

Paper Code: BSc-H-Math-501 **A** **SVSU:2023-24/R)**

Enrollment No.														

Program Name: B.Sc.-H-Math

V Semester / III Year Examination

Subject Name: Metric Spaces

[Time : 3:00 Hours] **[Max. Marks:100]**

I Objectives Question: Answer All Question **[15×2=30]**

1. Let d be a metric on x then the symmetry property of d is
(a) $d(x,y) \geq 0$ (b) $d(x,y) = d(y,x)$
(c) $d(x,y) = 0$ if and only if $x=y$ (d) none of these

2. In interval $[a, b]$ the usual metric space
(a) open set (b) closed set (c) Half open set (d) Half closed set

3. The usual metric d on R is given by
(a) $d(x,y) = x+y$ (b) $d(x,y) = |x-y|$ (c) $d(x,y) = |x+y|$ (d) none of these

4. A proper subset of countable set is
(a) uncountable (b) countable (c) Infinite (d) none of these

5. The Set A is countable if A is
(a) Finite (b) Denumerable
(c) Either finite or Denumerable (d) none of these

6 In a metric space (X,d)
(a) $d(x,y) = -d(y,x)$ (b) $d(x,y) = 2d(y,x)$ (c) $d(x,y) = d(y,x)$ (d) none of these

7 If (X,d) be a metric space and $d(x,y) = |x^2 - y^2|$ then d is
(a) usual metric space (b) discrete metric space
(c) pseudo metric space (d) none of these

8 A set S is open if
(a) it is not closed (b) it contains nbd of each of its point
(c) it does not contain nbd of any of its point (d) none of these

9 Let A be a set and D(A) is derived set of A, then A is closed set if
(a) D(A) is a subset of A (b) D(A) is not a subset of A
(c) D(A)=A (d) none of these

10 The union of open intervals is
(a) closed set (b) empty set (c) open set (d) none of these

11 A set S is nbd of point p if
(a) $]p, p+\varepsilon[$ (b) $]p-\varepsilon, p[$ (c) $]p-\varepsilon, p+\varepsilon[$ (d) none of these

12 The set of all boundary points of A is
(a) exterior of A (b) interior of (c) frontier (d) boundary of A

13 exterior of set A

(a) the set of all exterior points (b) the set of all boundary points
(c) the set of all interior points (d) none of these

14 The usual metric space (R,d) is

(a) not compact (b) compact
(c) A and B both are true (d) none of these

15 Every compact metric space is

(a) complete (b) not complete
(c) A and B both are true (d) none of these

II. Long Question: Answer any Three **[15×3=45]**

1. Let R be the set of all real numbers and let
   d(x,y) $= \frac{|x-y|}{1+|x-y|} \forall x, y \in$ R Show that d is metric for R
2. In a metric space ,every closed sphere is a closed
3. Let (X,d) be complete metric space , and let Y be a sub space of X then Y is complete if and only if Y is closed
4. A metric space (X,d) is disconnected if and only if there exists a non-empty Proper subset of X which is both d-open and d-closed.
5. Show that every compact subset A of metric space X is closed

III. Short Question: Answer any Five **[5×5=25]**

1. Define continuity of metric space
2. Define compactness of metric space
3. Show that every convergent sequence in a metric space is Cauchysequence
4. Show that on a real line every open interval is an open set
5. Define open set and closed set on a metric space
6. Give two different metrics for the set R of real numbers
7. if R denote the set of real numbers The function d: RXR → R defined d(x,y) =$|x-y|$,∀ x,y∈ R is a metric on R
8. Let d_1 and d_2 two metric space on X then d, defined by d(x,y) = d_1(x, y) +d_2(x,y) is also a metric space for X
9. Define Pseudo -metric
10. Define continuity of metric space

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Printing Pages :3

Paper Code :- BSc-H-Math-503 B (SVSU:2022-23/R)

Enrollment No.														

Program Name B.Sc. (H)

Semester/ Year: V/III

Subject Code: B.SC H Math 503

Subject Name: Numerical Analysis

[Time : 03:00 Hours] **[Max. Marks : 100]**

Note : 1. Attempt all the questions as per given instructions.

1. Multiple Choice Questions (MCQs) **[02×15=30]**

I In LU decomposition method ,if A = LU , then L is a … …
(a) column matrix (b) lower triangular matrix
(c) diagonal matrix (d) upper triangular matrix.

II The convergence of which of the following method depends on initial assumed value?
a) False position b) Gauss Seidel method
c) Newton Raphson method d) Euler method

III In the general Newton-Raphson formula is
(a) $x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$ (b) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$
(c) $x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$ (d) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

IV Gauss Seidal method is also termed as a method of _______
a) Successive displacement b) Eliminations
c) False positions d) Iterations

V Newton- Gregory Forward interpolation formula can be used ___
a) only for equally spaced intervals
b) only for unequally spaced intervals
c) for both equally and unequally spaced intervals
d) for unequally intervals

VI What is the percentage decrease in an interval containing root after iteration is applied by Bisection Method?
a) 20% b) 30% c) 40% d) 50%

VII The value of $\Delta f(x)$ *is* , where Δ is forward difference operator and h is interval
(a) f (x +h) + f(x) (b) f (x − h) − f(x)
(c) f (x +h) − f(x) (d) f (x) − f(x − h)

VIII The Lagrange's interpolation formula is used for.........................spaced value of x.
(a) equal (b) unequal
(c) neither (a) and nor (b) (d) both(a) and (b)

IX The n^{th} order divided differences $f(x_0, x_1, x_2 \dots \dots \dots x_n)$ for the function $f(x) = \frac{1}{x}$., is equal to:

(a) $\frac{(1)^n}{x_0 x_1 \dots \dots x_n}$ (b) $\frac{(-1)^{n-1}}{x_0 x_1 \dots \dots x_n}$

(c) $\frac{(-1)^{n+1}}{x_0 x_1 \dots \dots x_n}$ (d) $\frac{(-1)^n}{x_0 x_1 \dots \dots x_n}$

X The first divided differences $f(x)$ for the argument x_0, x_1 is

(a) $\frac{f(x_1) - f(x_0)}{x_1 - x_0}$ (b) $\frac{f(x_1) + f(x_0)}{x_1 + x_0}$ (c) $\frac{f(x_1) - f(x_0)}{x_1 + x_0}$

(d) $\frac{f(x_1) + f(x_0)}{x_1 - x_0}$

XI Find n if $x_0 = 0.75825$, x = 0.759 and h = 0.00005.

a) 1.5
b) 15
c) 2.5
d) 25

XII The convergence of which of the following method depends on initial assumed value?

a) False position
b) Gauss Seidel method
c) Newton Raphson method
d) Euler method

XIII The equation f(x) is given as $x^3 - x^2 + 4x - 4 = 0$. Considering the initial approximation at x=2 then the value of next approximation correct upto 2 decimal places is given as ________

a) 0.67
b) 1.33
c) 1.00
d) 1.50

XIV In Simpson's one third(3/8) rule, formula can be used if

(a) n is even
(b) n is in an integer
(c) n is the multiple of 3
(d) n is the multiple of 6

XV Which of the following method not convergent?

a) False position b) Bisection method
c) Newton Raphson method d) None of these

2. Answer in long (any three) **[15×3=45]**

I From the following table gives the population of a town during the last six censuses. Estimate using any suitable interpolation formula, the increase in the population during the period from 1946 to 1948.

Year	1911	1921	1931	1941	1951	1961
Population	12	15	20	27	39	52

II The equation $f(x) = x^6 - x^4 - x^3 - 1$ has a real root between 1.5 and 1.4 find the root upto 2 decimal point by using the regula-falsi method.

III Find the first, second and third derivatives of the function tabulated below, at the point $x = 1.5$

x	1.5	2	2.5	3.0	3.5	4.0
f(x)	3.375	7.000	13.625	24	38.875	59

IV Find the solution of the system of equations by gauss seidel method

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

3. Answer in short (any five) **[5x5=25]**

I Find $\int_2^{10} \frac{dx}{1+x}$ by using Simpson's 1/3 rule by dividing the range into 8 equal parts.

II Find the real root of the equation by using the $f(x) = x^3 - x - 1$, bysection method.

III Use Newton's method to find a root of the equation $f(x) = x^3 - 3x - 5$, which is nearer to x=3, correct to two decimals.

IV Find the LU decomposition of the following. $\begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

V A curve is drawn to pass through the points given by the following table:

x	1	1.5	2	2.5	3.0	3.5	4.0
f(x)	2	2.4	2.7	2.8	3.0	2.6	2.1

Find the area bounded by the curve, the x-axis and the lines x=1, x=4

VI For the following table find the form of the find the form of the function f(x) by using the Lagrange's formula.

x	0	1	2	5
f(x)	2	3	12	147

VII (i) Write down Lagrange interpolation formula.
(ii)Write down the Newton-Raphson formula.